LESSON 12

Quadratic Formula

A ***quadratic*** is an equation that has an unknown or variable raised to the second power, as in Y^2 or A^2. In factoring and in completing the square, we have been dealing exclusively with quadratic equations. So far, we can find the solution to a quadratic equation by factoring it, or if this fails, by completing the square. In this lesson we are going to complete the square with variables in order to discover a formula to solve all quadratics. If you've mastered the previous lesson, try solving the following equation by completing the square, and then compare your solution with mine.

$$AX^2 + BX + C = 0$$

Divide by the coefficient of X^2.

$$\frac{AX^2}{A} + \frac{BX}{A} + \frac{C}{A} = 0$$

$$X^2 + \frac{BX}{A} + \frac{C}{A} = 0$$

Add the opposite of the third term to both sides.

$$X^2 + \frac{BX}{A} = -\frac{C}{A}$$

Take one-half of the coefficient of the middle term, square it, and add the result to both sides.

$$X^2 + \frac{BX}{A} + \left(\frac{B}{2A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{2A}\right)^2$$

Factor the left side.

$$\left(X + \frac{B}{2A} \right)^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

Combine terms on the right.

$$\left(X + \frac{B}{2A} \right)^2 = -\frac{4AC}{4A^2} + \frac{B^2}{4A^2}$$

Take the square root of both sides.

$$X + \frac{B}{2A} = \sqrt{-\frac{4AC}{4A^2} + \frac{B^2}{4A^2}} = \pm\frac{\sqrt{-4AC + B^2}}{2A}$$

Subtract B/2A from both sides, and combine.

$$X = -\frac{B}{2A} \pm \frac{\sqrt{-4AC + B^2}}{2A}$$

The ***quadratic formula***! This is the form in which it is usually written.

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 1

Let's try an equation that we can answer by factoring, and "plug in" the values for A, B, and C. Remember that to find A, B, and C, the equation must be in the form $AX^2 + BX + C = 0$.

$$X^2 + 5X + 6 = 0$$

A = 1, B = 5, and C = 6

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$X = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$X = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

We can also solve $X^2 + 5X + 6 = 0$ by factoring.

$$X^2 + 5X + 6 = 0$$
$$(X + 2)(X + 3) = 0$$
$$X + 2 = 0 \quad X + 3 = 0$$
$$X = -2 \qquad X = -3$$

For this problem, it would have much easier to solve by factoring. Try factoring first, and if it doesn't work, use the quadratic formula. Here is another problem to try.

Example 2

Find the factors of $2X^2 = -7X - 4$.

To find A, B, and C, the equation must be in the form $AX^2 + BX + C = 0$.

$$2X^2 + 7X + 4 = 0$$

$A = 2$, $B = 7$, and $C = 4$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$
$$X = \frac{-7 \pm \sqrt{49 - 32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$$
$$X = \frac{-7 \pm \sqrt{17}}{4}$$
$$X = \frac{-7 + \sqrt{17}}{4} \text{ or } \frac{-7 - \sqrt{17}}{4}$$

Practice Problems 1

Solve for X. Try factoring first, and then use the quadratic formula if necessary.

1. $X^2 - 25 = 0$
2. $X^2 - 18X = -81$
3. $2X^2 + 7X + 6 = 0$
4. $3X^2 + X - 4 = 0$
5. $4A^2 - 36 = 0$
6. $X^2 + 5 = -3X$
7. $7X^2 = -2X + 1$
8. $2X^2 + 2X - 5 = 0$
9. $\frac{5}{X+3} + \frac{2}{X-3} = 5 \quad (X \neq \pm 3)$
10. $4X^2 = 9$
11. $4X^2 + 20X = -25$
12. $3Q^2 = -4Q - 2$

Solutions 1

1. $(X+5)(X-5) = 0$

   $X + 5 = 0 \quad X - 5 = 0$

   $X = -5 \quad X = 5$

2. $(X-9)(X-9) = 0$

   $X - 9 = 0 \quad X - 9 = 0$

   $X = 9 \quad X = 9$

3. $(2X+3)(X+2) = 0$

   $2X + 3 = 0 \quad X + 2 = 0$

   $2X = -3$

   $X = -3/2 \quad X = -2$

4. $(3X+4)(X-1) = 0$

   $3X + 4 = 0 \quad X - 1 = 0$

   $3X = -4$

   $X = -4/3 \quad X = 1$

5. $(2A-6)(2A+6) = 0$

   $2A - 6 = 0 \quad 2A + 6 = 0$

   $2A = 6 \quad 2A = -6$

   $A = 6/2 \quad A = -6/2$

   $A = 3 \quad A = -3$

6. $X = \dfrac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$X = \dfrac{-3 \pm \sqrt{-11}}{2} = \dfrac{-3 + i\sqrt{11}}{2}$ or $X = \dfrac{-3 - i\sqrt{11}}{2}$

7. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot -1}}{2 \cdot 7} = \dfrac{-2 \pm 4\sqrt{2}}{14} = \dfrac{-1 + 2\sqrt{2}}{7}$ or $X = \dfrac{-1 - 2\sqrt{2}}{7}$

8. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2} = \dfrac{-2 \pm 2\sqrt{11}}{4} = \dfrac{-1 + \sqrt{11}}{2}$ or $X = \dfrac{-1 - \sqrt{11}}{2}$

9. $\dfrac{5}{X+3} + \dfrac{2}{X-3} = 5$

$5(X-3) + 2(X+3) = 5(X^2 - 9)$

$7X - 9 = 5X^2 - 45$

$5X^2 - 7X - 36 = 0$

$X = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 5 \cdot -36}}{2 \cdot 5}$

$X = \dfrac{7 \pm \sqrt{769}}{10}$

$X = \dfrac{7 + \sqrt{769}}{10}$ or $X = \dfrac{7 - \sqrt{769}}{10}$

10. $(2X-3)(2X+3) = 0$

$2X - 3 = 0$ $\quad$ $2X + 3 = 0$

$2X = 3$ $\quad$ $2X = -3$

$X = 3/2$ $\quad$ $X = -3/2$

11. $(2X+5)(2X+5) = 0$

$2X + 5 = 0$ $\quad$ $2X + 5 = 0$

$2X = -5$ $\quad$ $2X = -5$

$X = -5/2$ $\quad$ $X = -5/2$

12. $3Q^2 + 4Q + 2 = 0$ $\quad$ $X = \dfrac{-(4) \pm \sqrt{(4)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$

$X = \dfrac{-4 \pm \sqrt{16 - 24}}{2 \cdot 3} = \dfrac{-4 \pm \sqrt{-8}}{2 \cdot 3}$

$X = \dfrac{-4 \pm i\sqrt{2 \cdot 4}}{2 \cdot 3} = \dfrac{-4 \pm 2i\sqrt{2}}{2 \cdot 3}$

$X = \dfrac{-2 + i\sqrt{2}}{3}$ or $\dfrac{-2 - i\sqrt{2}}{3}$

LESSON PRACTICE

Find the roots, using the quadratic formula when necessary.

1. $X^2 + 6X + 2 = 0$

2. $X^2 - 5X + 4 = 0$

3. $3X^2 + 7X - 1 = 0$

4. $A^2 - 10A = 11$

5. $2Q^2 + 2 = 17Q$

6. $5X^2 + 15X + 10 = 0$

7. $1/4\ R^2 - 1/2\ R + 3/2 = 0$

8. $16X^2 = 2X + 4$

9. $2X^2 + 3X - 8 = 0$

10. $Y^2 = 3/4\ Y + 2$

LESSON PRACTICE

Find the roots, using the quadratic formula when necessary.

1. $8X^2 - X - 3 = 0$

2. $7 = 2X^2 + X$

3. $Q^2 - 6Q + 3 = 0$

4. $2 + 3X + 4X^2 = 0$

5. $P = P^2 - 2$

6. $X^2 + 1/5\ X + 5 = 0$

7. $20X^2 + 40X = 30$

8. $5A^2 + 2A - 1 = 0$

9. $3X^2 = -5X$

10. $AX^2 + BX + C = 0$

Find the roots, using the quadratic formula when necessary.

1. $X^2 - 5X + 6 = 0$

2. $X^2 + 4X + 2 = 0$

3. $X^2 - 3X + 1 = -6X$

4. $X^2 + 4X - 12 = 0$

5. $2X^2 + 2X + 5 = 0$

6. $X^2 + 8X = -16$

Complete the square.

7. $X^2 - 26X +$ _____

8. $2X^2 + 9X +$ _____

9. $X^2 +$ _____ $+ 400$

10. $X^2 -$ _____ $+ 14$

Solve for X. Complete the square when necessary.

11. $X^2 + 1/3\ X - 4/3 = 0$

12. Check the answers to #11 by placing them in the original equation.

13. Expand $(X - A)^6$.

14. What is the second term of $(1/2\ X - 3A)^4$?

15. Expand $(5 - 2A)^3$.

16. Find the cube root of $X^3 - 6X^2Y + 12XY^2 - 8Y^3$.

Put in standard form.

17. $\dfrac{6 + 5i}{3i - 2}$

18. $\dfrac{2 + \sqrt{-49}}{2 - \sqrt{-49}}$

Simplify, and combine like terms when possible.

19. $\dfrac{2}{3 - \sqrt{7}}$

20. $\dfrac{2 + \sqrt{5}}{2\sqrt{5} - 4}$

Find the roots, using the quadratic formula when necessary.

1. $2X^2 - 9X - 7 = 0$
2. $X^2 + 5X - 2 = 0$
3. $3X^2 + 7X + 4 = 0$
4. $X^2 - 6X + 12 = 0$
5. $5X^2 - 3X - 2 = 0$
6. $4X^2 + 1 = 4X$

Complete the square.

7. $X^2 + 5X + \_\_\_\_$
8. $X^2 - 1/2\ X + \_\_\_\_$
9. $25X^2 + \_\_\_\_ + 1$
10. $49X^2 - \_\_\_\_ + 4$

Solve for X. Complete the square when necessary.

11. $X^2 - 12X + 20 = 0$
12. Check the answers to #11 by placing them in the original equation.

13. Expand $(X + 1)^4$.

14. What is the fifth term of $(1/2\ X - 3A)^4$?

15. Expand $(10 - 1/X)^3$.

16. Find the cube root of $X^3 + 6X^2 + 12X + 8$.

Put in standard form.

17. $\dfrac{4 - 3i}{2i}$

18. $\dfrac{10 + \sqrt{-A}}{10 - \sqrt{-A}}$

Simplify, and combine like terms when possible.

19. $\dfrac{9}{7 + \sqrt{10}}$

20. $\dfrac{4 - \sqrt{6}}{3\sqrt{7} + 5}$

TEST

Circle your answer.

1. Which of the following cannot be solved using the quadratic equation?

   A. $X^2 - 64 = 0$
   B. $X^3 + 3Y + 1 = 0$
   C. $4A^2 + 8A = 16$
   D. $Y^2 = 2Y + 4$

2. The part of the quadratic formula written under the radical is:

   A. $B^2 + 4AC$
   B. $B^2 - 4AC$
   C. $-B^2 \pm 4AC$
   D. $A^2 + 4BC$

3. All quadratic equations can be solved by:

   A. factoring
   B. both factoring and the quadratic formula
   C. the quadratic formula
   D. none of the above

4. In order to find values of A, B, and C in the quadratic formula, an equation should be in the form:

   A. $AX^2 = BX + C$
   B. $X^2 + AX = B - C$
   C. $AX^2 + BX + C = 0$
   D. $AX^2 + BX = -C$

5. The solution to $7X^2 + 2X - 1 = 0$ can be written as:

   A. $X = \frac{-2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$

   B. $X = \frac{2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$

   C. $X = \frac{-2 \pm \sqrt{2^2 + (4)(7)(-1)}}{2(7)}$

   D. $X = \frac{-2 \pm \sqrt{(-2)^2 - (4)(7)(-1)}}{2}$

For #6–10, solve using the best method.

6. $X^2 - 36 = 0$

   A. $X = 6, -6$
   B. $X = 4, 9$
   C. $X = 0, 6$
   D. $X = \pm 9$

7. $X^2 + 3 = -3X$

   A. $X = \frac{-3 \pm \sqrt{3}}{2}$

   B. $X = \frac{-3 \pm i\sqrt{3}}{6}$

   C. $X = \frac{3 \pm i\sqrt{3}}{2}$

   D. $X = \frac{-3 \pm i\sqrt{3}}{2}$

8. $5X^2 = -2X + 1$

   A. $X = \frac{-1 \pm \sqrt{5}}{5}$

   B. $X = \frac{-1 \pm \sqrt{6}}{5}$

   C. $X = \frac{1 \pm 2\sqrt{6}}{5}$

   D. $X = \frac{1 \pm \sqrt{5}}{5}$

9. $4X^2 + 20X = -25$

   A. $X = \pm 5/2$
   B. $X = 4, 5$
   C. $X = 5/2$
   D. $X = -5/2$

10. $4X^2 + 4X - 10 = 0$

   A. $X = \frac{-1 \pm i\sqrt{11}}{2}$

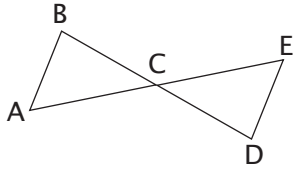
   B. $X = i, -2i$

   C. $X = \frac{-1 \pm \sqrt{11}}{2}$

   D. $X = \frac{-1 \pm 3i}{2}$

11. ΔABC is congruent to ΔEDC. $\overline{AB}$ corresponds to:

   A. $\overline{BA}$
   B. $\overline{AC}$
   C. $\overline{ED}$
   D. $\overline{BC}$



12. A quadrilateral with only one pair of parallel sides is a:

   A. rhombus
   B. trapezoid
   C. parallelogram
   D. regular polygon

13. Two sides of triangle A are congruent to the corresponding sides of triangle B. The angle formed by the corresponding sides is 25° in both triangles. What postulate may be used to prove triangles A and B congruent?

   A. SSS
   B. SSA
   C. SAS
   D. cannot be proved congruent

14. Each angle of triangle ABC is congruent to the corresponding angle of triangle DEF. What postulate may be used to prove ΔABC and ΔDEF congruent?

   A. SSS
   B. AAA
   C. SAS
   D. cannot be proved congruent

15. Five yards are a little less than:

   A. 5 meters
   B. 10 meters
   C. 2 meters
   D. 6 meters

19. $(18i)\left(\sqrt{-36}+7i\right) = (18i)(6i+7i) =$
$(18i)(13i) = 234i^2 = 234(-1) = -234$

20. $\left(i^2\right)\left(i^1\right)\left(i^3\right) = i^{2+1+3} = i^6 = (-1)^3 = -1$

Lesson Practice 12A

1. $X = \dfrac{-(6) \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)} = \dfrac{-6 \pm \sqrt{28}}{2} =$
$\dfrac{-6 \pm 2\sqrt{7}}{2} = -3 \pm 2\sqrt{7}$

2. $X^2 - 5X + 4 = 0$
$(X-4)(X-1) = 0$

$X - 4 = 0$ $\qquad$ $X - 1 = 0$
$X = 4$ $\qquad$ $X = 1$

3. $X = \dfrac{-(7) \pm \sqrt{(7)^2 - 4(3)(-1)}}{2(3)} = \dfrac{-7 \pm \sqrt{61}}{6}$

4. $A^2 - 10A - 11 = 0$
$(A-11)(A+1) = 0$

$A - 11 = 0$ $\qquad$ $A + 1 = 0$
$A = 11$ $\qquad$ $A = -1$

5. $2Q^2 + 2 = 17Q$
$2Q^2 - 17Q + 2 = 0$
$\dfrac{-(-17) \pm \sqrt{(-17)^2 - 4(2)(2)}}{2(2)} = \dfrac{17 \pm \sqrt{273}}{4}$

6. $5X^2 + 15X + 10 = 0$
$(5)(X+1)(X+2) = 0$

$X + 1 = 0$ $\qquad$ $X + 2 = 0$
$X = -1$ $\qquad$ $X = -2$

7. $\dfrac{1}{4}R^2 - \dfrac{1}{2}R + \dfrac{3}{2} = 0$
$(4)\dfrac{1}{4}R^2 - (4)\dfrac{1}{2}R + (4)\dfrac{3}{2} = (4)0$
$R^2 - 2R + 6 = 0$
$\dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(6)}}{2(1)} =$
$\dfrac{2 \pm \sqrt{-20}}{2} = \dfrac{2 \pm 2i\sqrt{5}}{2} = 1 \pm i\sqrt{5}$

8. $16X^2 = 2X + 4$
$8X^2 = X + 2$
$8X^2 - X - 2 = 0$
$X = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-2)}}{2(8)} = \dfrac{1 \pm \sqrt{65}}{16}$

9. $X = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(2)(-8)}}{2(2)} = \dfrac{-3 \pm \sqrt{73}}{4}$

10. $Y^2 = \dfrac{3}{4}Y + 2$
$(4)Y^2 = (4)\dfrac{3}{4}Y + (4)2$
$4Y^2 = 3Y + 8$
$4Y^2 - 3Y - 8 = 0$
$X = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-8)}}{2(4)} = \dfrac{3 \pm \sqrt{137}}{8}$

Lesson Practice 12B

1. $X = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-3)}}{2(8)} = \dfrac{1 \pm \sqrt{97}}{16}$

2. $7 = 2X^2 + X$
$0 = 2X^2 + X - 7$
$X = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(2)(-7)}}{2(2)} = \dfrac{-1 \pm \sqrt{57}}{4}$

3. $Q = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \dfrac{6 \pm \sqrt{24}}{2} =$
$\dfrac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$

4. $2 + 3X + 4X^2 = 0$
$4X^2 + 3X + 2 = 0$
$X = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(4)(2)}}{2(4)} =$
$\dfrac{-3 \pm \sqrt{-23}}{8} = \dfrac{-3 \pm i\sqrt{23}}{8}$

5. $P = P^2 - 2$
$0 = P^2 - P - 2$
$0 = (P-2)(P+1)$

$P - 2 = 0$ $\qquad$ $P + 1 = 0$
$P = 2$ $\qquad$ $P = -1$