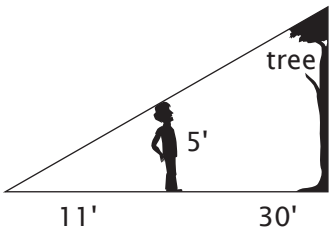
LESSON 6

Angles of Elevation and Depression

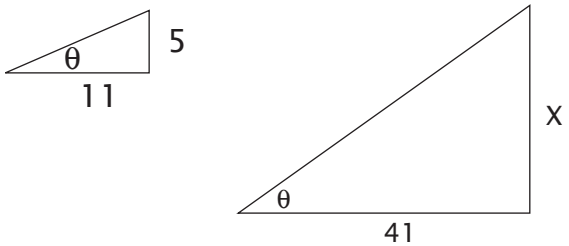
Now we get a chance to apply all of our newly acquired skills to real-life applications, otherwise known as word problems. Let's look at some elevation and depression problems. I first encountered these in a Boy Scout handbook many years ago. There was a picture of a tree, a boy, and several lines.

Example 1



How tall is the tree?

Separating the picture into two triangles helps to clarify our ratios.



We could write this as a proportion (two ratios), $\frac{5}{11} = \frac{X}{41}$, and solve for X.

We can also use our trig abilities.

From the "boy" triangle: $\tan\theta = \frac{5}{11} = .4545$ $\theta = 24.44°$

From the large triangle: $\tan 24.44° = \frac{X}{41}$

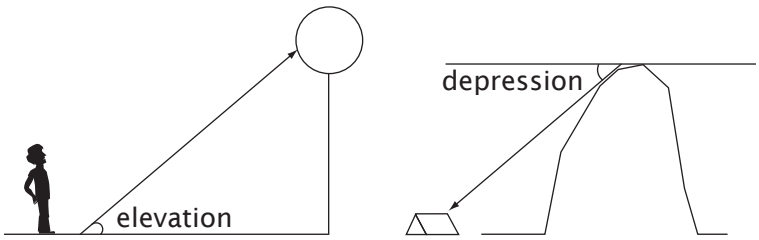
Solve for X. $(41)(.4545) = X$

$18.63 = X$ The tree is 18.63'.

When working these problems, the value of the trig ratio may be rounded and recorded, and further calculations made on the rounded value. You may also keep the value of the ratio in your calculator and continue without rounding the intermediate step. This may yield slightly different final answers. These differences are not significant for the purposes of this course.

It is pretty obvious that an ***angle of elevation*** measures up and an ***angle of depression*** measures down. One of the keys to being a good problem solver is to draw a picture using all the data given. It turns a one-dimensional group of words into a two-dimensional picture.

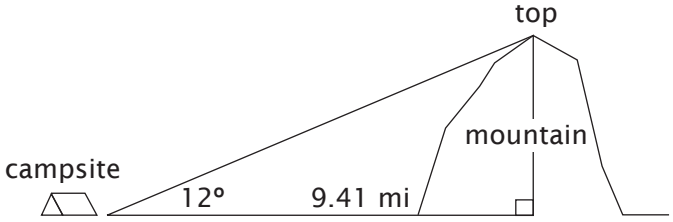
Figure 1



We assume that the line where the angle begins is perfectly flat or horizontal.

Example 2

A campsite is 9.41 miles from a point directly below the mountain top. If the angle of elevation is 12° from the camp to the top of the mountain, how high is the mountain?



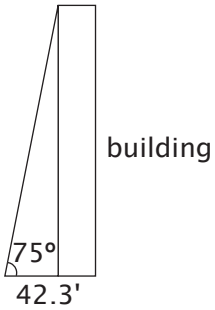
You can see a right triangle with the side adjacent to the 12° angle measuring 9.41 miles. To find the height of the mountain, or the side opposite the 12° angle, the tangent is the best choice.

$$\tan 12° = \frac{\text{height}}{9.41 \text{ mi}}$$
$$(9.41)(\tan 12°) = \text{height}$$
$$(9.41)(.2126) = \text{height}$$
$$2 \text{ miles} = \text{height}$$

Example 2

At a point 42.3 feet from the base of a building, the angle of elevation of the top is 75°. How tall is the building?

$$\tan 75° = \frac{\text{height}}{42.3'}$$
$$(42.3)(\tan 75°) = \text{height}$$
$$(42.3)(3.7321) = \text{height}$$
$$157.87' = \text{height of the building}$$

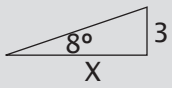


Practice Problems 1

1. How far from the door must a ramp begin in order to rise three feet with an 8° angle of elevation?

2. An A-frame cabin is 26.23 feet high at the center, and the angle the roof makes with the base is 53°15'. How wide is the base?

Solutions 1

1. 

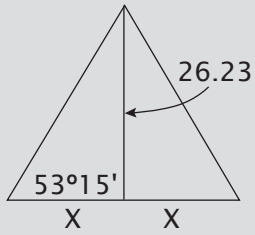
$$\tan 8° = \frac{3}{X}$$

$$X \tan 8° = 3$$

$$X = \frac{3}{\tan 8°}$$

$$X = \frac{3}{.1405}$$

$$X = 21.35 \text{ ft}$$

2. 

$$53°15'' = 53.25°$$

$$\tan 53.25° = \frac{26.23}{X}$$

$$X = \frac{26.23}{\tan 53.25°}$$

$$X = \frac{23.26}{1.3392}$$

$$X = \frac{26.23}{1.3392}$$

$$X = 19.59$$

$$2X = 39.18 \text{ ft}$$

Answer the questions.

1. Isaac's camp is 5,280 feet from a point directly beneath Mt. Monadnock. What is the hiking distance along the ridge if the angle of elevation is 25° 16'?

2. How many feet higher is the top of the mountain than his campsite?

Express as a fraction.

3. csc θ =

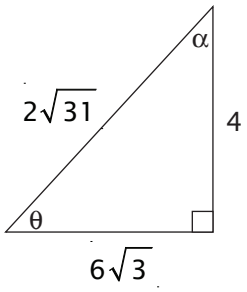
4. sec θ =

5. cot θ =

6. csc α =

7. sec α =

8. cot α =



Express as a decimal.

9. sin θ =

10. cos θ =

11. tan θ =

12. sin α =

13. cos α =

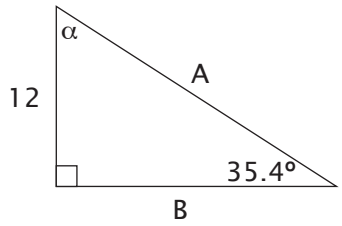
14. tan α =

15. Use your answers from #9–11 to find the measure of θ.

16. Use your answers from #12–14 to find the measure of α.

Solve for the lengths of the sides and the measures of the angles.

17.



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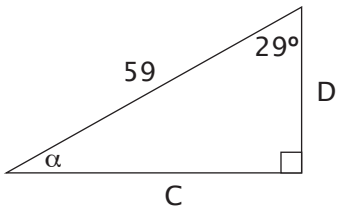

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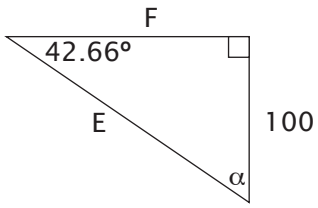

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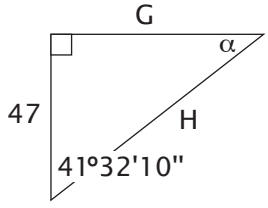


Answer the questions.

1. The side of a lake has a uniform angle of elevation of 15° 30'. How far up the side of the lake does the water rise if, during the flood season, the height of the lake increases by 7.3 feet?

2. A building casts a shadow of 110 feet. If the angle of elevation from that point to the top of the building is 29° 3', find the height of the building.

Express as a fraction.

3. csc θ =

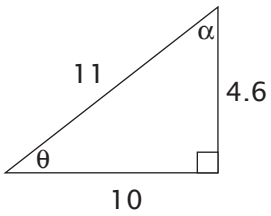
4. sec θ =

5. cot θ =

6. csc α =

7. sec α =

8. cot α =



Express as a decimal.

9. sin θ =

10. cos θ =

11. tan θ =

12. sin α =

13. cos α =

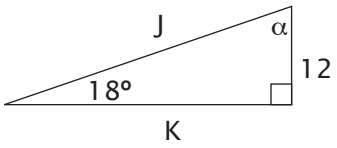
14. tan α =

15. Use your answers from #9–11 to find the measure of θ.

16. Use your answers from #12–14 to find the measure of α.

Solve for the lengths of the sides and the measures of the angles.

17.



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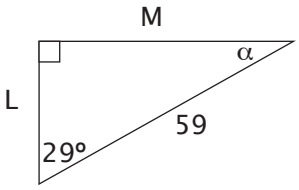

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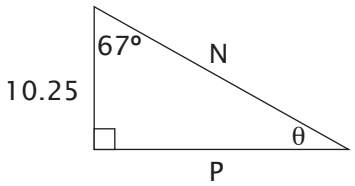

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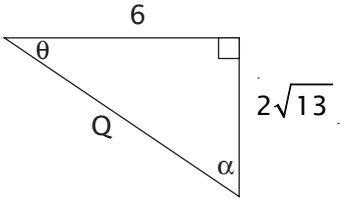


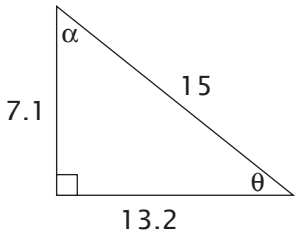
Answer the questions.

1. From a point 120 feet from the base of a church, the angles of elevation of the top of the building and the top of a cross on the building are 38° and 43° respectively. Find the height to the top of the cross. (The ground is flat.)

2. Find the height of the building as well as the height of the cross by itself.

Express as a fraction.

3. csc θ =
4. sec θ =
5. cot θ =
6. csc α =
7. sec α =
8. cot α =



Express as a decimal.

9. sin θ =
10. cos θ =
11. tan θ =
12. sin α =
13. cos α =
14. tan α =

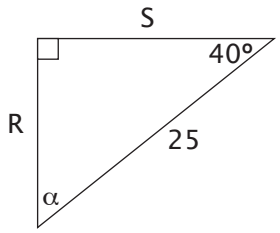
Results for #15 and 16 may vary slightly from the solutions, depending on when steps were rounded.

15. Use your answers from #9–11 to find the measure of θ.

16. Use your answers from #12–14 to find the measure of α.

Solve for the lengths of the sides and the measures of the angles.

17.



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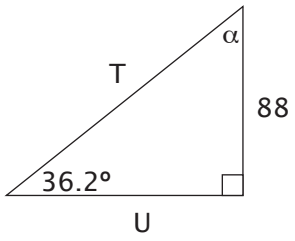

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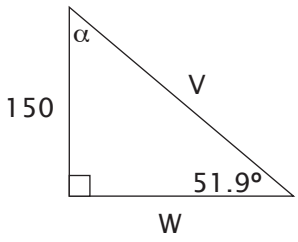

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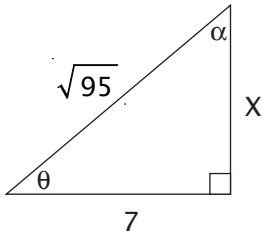


Answer the questions.

1. A campsite is 12.88 miles from a point directly below Mt. Adams. If the angle of elevation is 15.5° from the camp to the top of the mountain, how high is the mountain?

2. At a point 60.7 feet from the base of a building, the angle of elevation from that point to the top is 64.75°. How tall is the building?

Express as a fraction.

3. csc θ =

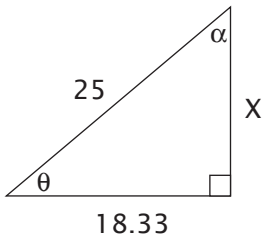
4. sec θ =

5. cot θ =

6. csc α =

7. sec α =

8. cot α =



Express as a decimal.

9. sin θ =

10. cos θ =

11. tan θ =

12. sin α =

13. cos α =

14. tan α =

15. Use your answers from #9–11 to find the measure of θ.

16. Use your answers from #12–14 to find the measure of α.

Solve for the lengths of the sides and the measures of the angles.

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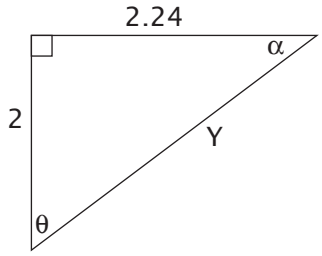

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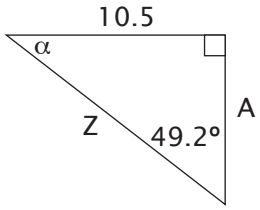

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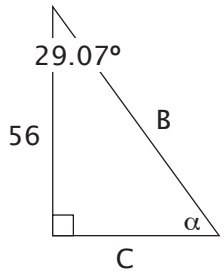

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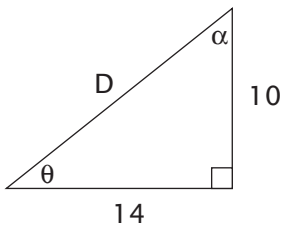


Here are some more applications of trig functions. In some of these you may need to find a missing side, and in others a missing angle.

Use the skills you have learned so far to answer the questions. Always begin by making a drawing and labeling the known information.

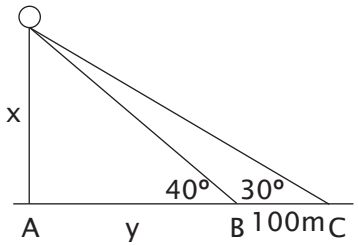
1. A girl who is 1.6 meters tall stands on level ground. The elevation of the sun is 60° above the horizon. What is the length of her shadow?

2. If the girl in #1 casts a shadow that is one meter long, what is the elevation of the sun?

3. A stairway forms an angle with the floor from which it rises. This angle may be called the angle of inclination. What is the angle of inclination of a stairway if the steps have a tread of 20 centimeters and a rise of 16 centimeters?

Some problems will require more of your algebra skills. There are some examples of these on the next page. The first one is done for you.

4. An observation balloon is attached to the ground at point A. On a level with A and in the same straight line, the points B and C were chosen so that BC equals 100 meters. From the points B and C, the angle of elevation of the balloon is 40° and 30° respectively. Find the height of the balloon.



First, make a drawing. There's not enough information to find x using either the angle at B or the angle at C.

However, we can make two equations using x and y.

Equation 1 $\tan 40° = \frac{x}{y}$ **Equation 2** $\tan 30° = \frac{x}{y+100}$

Replace tan 40° with its ratio and solve for x in Equation 1.

$$.8391 = \frac{x}{y} \text{ or } x = .8391y$$

Replace tan 30° with its ratio in in Equation 2. $.5774 = \frac{x}{y+100}$

Substitute value of x from Equation 1 in Equation 2. $.5774 = \frac{.8391y}{y+100}$

Solve for y.
$$\begin{aligned} .5774(y + 100) &= .8391y \\ .5774\,y + 57.74 &= .8391y \\ 57.74 &= .2617y \\ y &= 220.6 \text{ (rounded)} \end{aligned}$$

Solve for x, which is the height of the balloon. $x = .8391y$

$x = .8391\ (220.6) = 185.1$ m

5. Tom wished to find the width of a river. He observed a tree directly across the river on the opposite bank. The angle of elevation to the top of the tree was 32°. Then Tom moved directly back from the bank 50 meters and found that the angle of elevation to the top of the tree was 21°. What is the width of the river?

6. In the side of a hill that slopes upward at an angle of 32°, a tunnel is bored sloping downward at an angle of 12°15' from the horizontal. How far below the surface of the hill is a point 38 meters down the tunnel?

Use for #1–4: Devan stands 926 meters from a point directly below the peak of a mountain. The angle of elevation between Devan and the top of the mountain is 42°.

1. Which equation can be used to find the height of the mountain (x)?

   A. sin 42° = x/926
   B. tan 42° = 926/x
   C. cos 48° = 926/x
   D. tan 42° = x/926

2. What is the height of the mountain?

   A. 833.8 m
   B. 1,028.4 m
   C. 619.6 m
   D. 1,383.9 m

3. A tower 50 meters high is built on top of the mountain. What is the angle of elevation from Devan's position to the top of the tower? (Round decimal degrees to tenths.)

   A. 40° 14' 44"
   B. 43° 42'
   C. 57° 15'
   D. 46° 20' 08"

4. If a bird flew from Devan's position to the top of the mountain, how many meters would it travel?

   A. 408.4 m
   B. 1,246.1 m
   C. 1,383.9 m
   D. 1,280 m

Use for #5–8: From a point 80 meters from the base of a building to the top of the building, the angle of elevation is 51°. From the same point to the top of a flag staff on the building, the angle of elevation is 54°.

5. What equation can be used to find the combined height (y) of building and flagpole?

   A. y = 80 tan 51°
   B. y = 80 sin 54°
   C. y = 80 tan 54°
   D. $y = \frac{\tan 51°}{80}$

6. What is the height of the building alone?

   A. 98.8 m
   B. 110.1 m
   C. 64.8 m
   D. 58.1 m

7. What is the height of the flagpole alone?

   A. 15.1 m
   B. 45.3 m
   C. 4.2 m
   D. 11.3 m

8. How long must a cable be in order to stretch from the observation point to the top of the building?

   A. 102.9 m
   B. 127.1 m
   C. 136.1 m
   D. 50.3 m

Use for #9–10: A car traveled a distance of 100 feet up a ramp to a bridge. The angle of elevation of the ramp was 10°.

9. How high was the bridge above road level?

   A. 17.4 ft
   B. 98.5 ft
   C. 10 ft
   D. 100 ft

10. What is the actual distance from the beginning of the ramp to the base of the bridge?

   A. 575 ft
   B. 98.5 ft
   C. 89.4 ft
   D. 17.4 ft

11. $\frac{\sqrt{3}}{3}$ is the ratio for:

   A. cos 45°
   B. cos 30°
   C. tan 60°
   D. tan 30°

12. Arcsin .8192 =

   A. 1.22
   B. 35°
   C. 55°
   D. .9999

13. 46° 21' 02" =

   A. 46.21°
   B. 46.12°
   C. 46.35°
   D. 46.4°

14. $\frac{\sin \alpha}{\cos \alpha}$ is equal to:

   A. tan α
   B. cot α
   C. sec α
   D. csc α

15. $\frac{1}{\cos \alpha}$ is equal to:

   A. csc α
   B. sec α
   C. sin α
   D. cos α

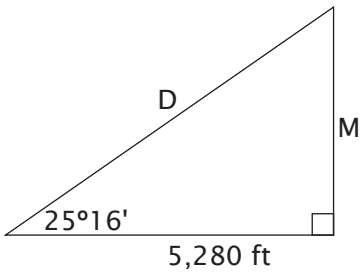
Lesson 6A

1. $\cos 25°16' = \dfrac{5{,}280}{D}$

$D\cos 25°16' = 5{,}280$

$D = \dfrac{5{,}280}{\cos 25°16'}$

$D \approx 5{,}838.77 \text{ ft}$



2. $\tan 25°16' = \dfrac{M}{5{,}280}$

$M = (5{,}280)(\tan 25°16')$

$M \approx 2{,}492.09 \text{ ft}$

3. $\csc\theta = \dfrac{2\sqrt{31}}{4} = \dfrac{\sqrt{31}}{2}$

4. $\sec\theta = \dfrac{2\sqrt{31}}{6\sqrt{3}} = \dfrac{\sqrt{31}}{3\sqrt{3}} = \dfrac{\sqrt{31}\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \dfrac{\sqrt{93}}{9}$

5. $\cot\theta = \dfrac{6\sqrt{3}}{4} = \dfrac{3\sqrt{3}}{2}$

6. $\csc\alpha = \dfrac{\sqrt{93}}{9}$

7. $\sec\alpha = \dfrac{\sqrt{31}}{2}$

8. $\cot\alpha = \dfrac{2\sqrt{3}}{9}$

9. $\sin\theta = \dfrac{4}{2\sqrt{31}} \approx .3592$

10. $\cos\theta = \dfrac{6\sqrt{3}}{2\sqrt{31}} \approx .9333$

11. $\tan\theta = \dfrac{4}{6\sqrt{3}} \approx .3849$

12. $\sin\alpha = \dfrac{6\sqrt{3}}{2\sqrt{31}} \approx .9333$

13. $\cos\alpha = \dfrac{4}{2\sqrt{31}} \approx .3592$

14. $\tan\alpha = \dfrac{6\sqrt{3}}{4} \approx 2.5981$

15. $\arcsin .3592 \approx 21.05°$

16. $\arcsin .9333 \approx 68.96°$

17. $\tan 54.6° = \dfrac{B}{12}$

$B = (12)(\tan 54.6°) \approx 16.89$

$\sin 35.4° = \dfrac{12}{A}$

$A\sin 35.4° = 12$

$A = \dfrac{12}{\sin 35.4°} \approx 20.72$

$\alpha = 90 - 35.4° = 54.6°$

18. $\sin 61° = \dfrac{D}{59}$

$D = (59)(\sin 61°) \approx 51.6$

$\cos 61° = \dfrac{C}{59}$

$C = (59)(\cos 61°) \approx 28.6$

$\alpha = 90° - 29° = 61°$

19. $\tan 47.34° = \dfrac{F}{100}$

$F = (100)(\tan 47.34°)$

$F \approx 108.52$

$\sin 42.66° = \dfrac{100}{E}$

$E\sin 42.66° = 100$

$E = \dfrac{100}{\sin 42.66°} \approx 147.57$

$\alpha = 90° - 42.66° = 47.34°$

20. $\tan 41.54° = \dfrac{G}{47}$

$G = (47)(\tan 41.54°) \approx 41.64$

$\cos 41.54° = \dfrac{47}{H}$

$H\cos 41.54° = 47$

$H = \dfrac{47}{\cos 41.54°} \approx 62.79$

$\alpha = 90° - 41°32'10'' = 48°27'50''$

$\theta \approx 41.54°$